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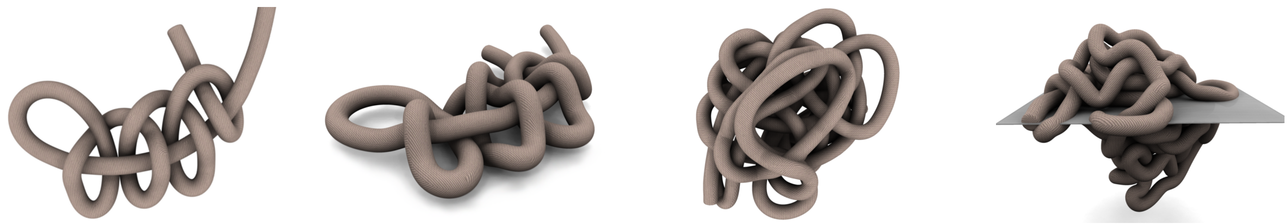

Sketch view | Side view | Sketch view | Side view with sketching plane

Figure 1: Examples of knots that are created using our sketch based interface.

Physical knots have always fascinated people. For instance, Ashley Book of Knots contains some 7000 illustrations covering over 2000 knots. Physical knots usually associated with sailing, but they are also used in weaving, knitting, braiding and crocheting. Mathematical knots are introduced to formally classify the physical knots [Turner and de Griend 1996]. The significant difference with mathematical and physical knots is that in mathematical knots threads are closed curves. This property helps to formulate the problem more precisely and several knot polynomials such as Alexander and Jones polynomials are introduced to categorize the knots. In decorative arts, the most well-known knot form is Celtic knots, which are also good examples that shows how difficult to draw for humans knots. To draw Celtic knots and to obtain cyclic plain weaving on surfaces mesh based methods are introduced [Kaplan and Cohen 2003; Akleman et al. 2009]. Except mesh based methods, we do not know any other way to provide people to design knots. There exist software such as Mathematica's knot theory package or Knotplot, but these software does not provide free form design of knots.

We present an unexpectedly easy to use interface to create knots by using sketch based modeling. In our interface the only thing the users need to do for creating knots and links is to draw a set of curves. These curves serves the medial axis of the knots to be constructed. To construct knots we first estimate of the depth –z– value for every point on the medial axis curve. The depth estimation turns 2D medial axis curve into a 3D medial axis. We then extrude a polygon along the 3D medial axis curve to obtain the tread that forms the physical knot. If the medial axis consists of closed curves, the result is a mathematical knot. The key part of our knot construction is the depth estimation. Our depth estimation is developed based on following observations:

1. The threads are drawn under orthographic transformation and they do not self intersect.
2. The threads cross other threads (or themselves) by alternatingly going over and under.

The first observation is natural and expected. The second one is not always desired but it significantly simplifies the interface and reduces the amount of intersection. For instance, The mathematical knots that exhibit this pattern is called alternating knots in mathematical knot theory. Since most prime knots are also alternating, our system automatically can create these knots from sketches. Alternating links such as well-known Borromean link can also be created automatically from sketches.

Many well-known physical knots can be created using alternating pattern. It is interesting to note that even if the knot is not alternating, it is still possible to obtain a desired knot by moving the treads closer. In this case, system counts two threads as one and it is possible to obtain desired knot automatically. There are also many objects around us from spaghetti to snakes whose visual appearance can be improved introducing some alternating in crossings. Weaving or knitting can also be obtained using such an approach. One issue is that a particular knot cannot be created by alternating pattern. To resolve those cases we provide an interface option to the user for reverse the order in a crossing by a mouse click.

References

Akleman, E., Chen, J., Xing, Q., and Gross, J. L. 2009. Cyclic plain-weaving on polygonal mesh surfaces with graph rotation systems. In *ACM SIGGRAPH 2009 papers*, SIGGRAPH '09, 78:1–78:8.

Kaplan, M., and Cohen, E. 2003. Computer generated celtic design. In *Proc. of 14th Eurographics Workshop on Rendering*, 9–16.

Turner, J., and de Griend, P. V. 1996. *History and Science of Knots*. World Scientific Publishing, Singapore.

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